

Mathematical Invariants in Nature and the Arts: Seeing, Discovering, Learning



by Pier Francesco Sciuto

Prologue

The Egyptians have a legend that the end of Osiris's life came on the seventeenth of the month, on which day it is quite evident to the eye that the period of the full moon is over. Because of this the Pythagoreans call this day "the Barrier", and utterly abominate this number. For the number seventeen, coming in between the square sixteen and the oblong rectangle eighteen, which, as it happens, are the only plane figures that have their perimeters equal to their areas, bars them off from each other and disjoins them, and breaks up the ratio of eight to eight and an eighth by its division into unequal intervals.

(Plutarch, *Isis and Osiris*)

Not those who begin, but those who persevere.

(Leonardo da Vinci)

To see a world in a grain of sand
And a heaven in a wild flower,
Hold infinity in the palm of your hand
And eternity in an hour.

(William Blake, *Auguries of Innocence*)

A true journey of discovery is not about seeking new lands but having new eyes.

(Marcel Proust, *In Search of Lost Time. The Captive*)

The joke is that the more prose-minded the scholar the more capable he is supposed to be of interpreting ancient poetic meaning, and that no scholar cares to set himself up as an authority than one narrow subject for fear of incurring the dislike and suspicion of his colleagues. To know only one thing well is to have a barbaric mind: civilization implies the graceful relation of all varieties of experience to a central humane system of thought. The present age is peculiarly barbaric: introduce, say, a hebrew scholar to an ichthyologist or an authority of danish place names and the pair of them would have non single topic in common but the weather or the war (if there happened to be a war in progress, which is usual in this barbaric age).

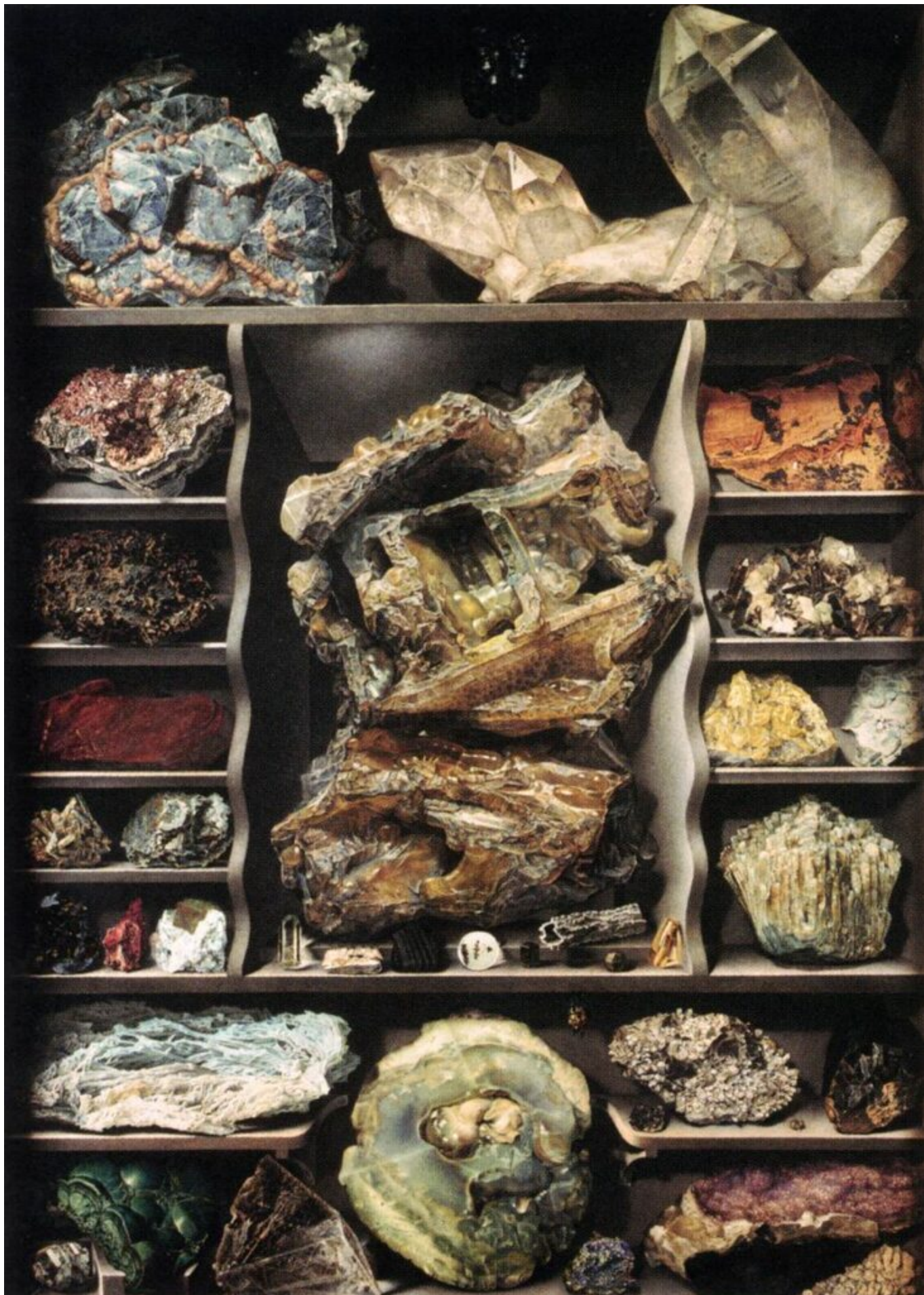
(Robert Graves, *The White Goddess*)

The series of lessons I begin presenting today on FD, of which this is intended to be a brief introduction, took shape over the course of a decade of activity as curator of the "Sandra Forni" Geological Garden Museum in Bologna. Interacting with teachers and questions from students, who took turns visiting the collection and participating in related excursions, gave rise to the first notes. The final version was written in 2020 during the lockdown due to the Covid-19 emergency.

The initial idea came from questions posed by teachers while discussing fossils. They wanted to know the equations relating to the shape of shells. At the time, I did not have any easily accessible bibliographical references, so I began to wonder how this gap could be filled. Similar situations subsequently arose in relation to various other topics. These topics ranged from the mathematical reproduction of crystal structures to general, speculative questions about the connection between art, nature, and mathematics. Mathematics has a profound connection with our surroundings, yet wonder often prevails over rational observation. In this context, underestimating the mathematical laws that govern creation is almost inevitable. This is why I decided to write this paper.

Since I had to link different topics together, I wanted to replicate the characteristics of a guided tour carried out in stages. I tried to be as conversational as possible and reduced explicitly mathematical dissertations. During the writing process, I consulted many texts, articles, and comments not mentioned here. Since the purpose is essentially informative, I will

limit bibliographical references to texts accessible to anyone interested in the relationship between science and art. I trust in the generosity of authors not cited.



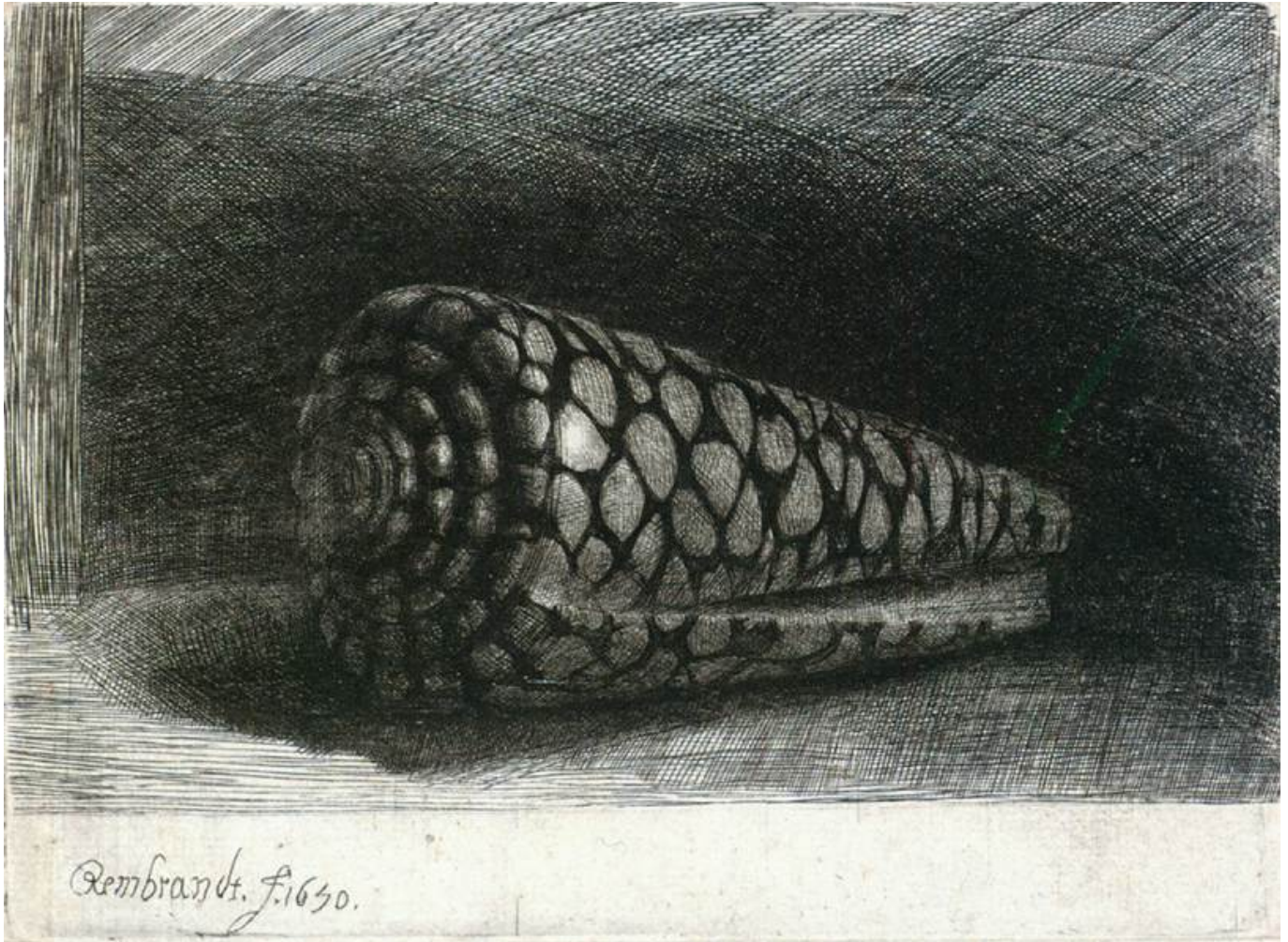
Alexandre-Isidore Leroy de Barde, *Crystallized Minerals*, early 19th century, watercolor and gouache on paper, 126 x 80 cm, Paris, Louvre (www.wga.hu).

The challenge

A walk on the beach can be more helpful for thinking and reflecting than spending many hours at your desk. Intellectual stimulation arises from the way we live our days. However, it can be challenging to find a concrete way to gather your thoughts when your attention wanders. When this happens, it helps to write down your thoughts before they slip away. My daughter gave me the opportunity to do so. A friend of hers had found a beautiful, large shell on the beach in Cervia, and we decided to look for one, too. I love collecting shells, one of the passions that led me to study geology. Many years ago, when studying for my paleontology exam, I got into the habit of collecting shells and reciting their names. Now, I was doing the same thing with my daughter. However, those shells were not what I expected.

The number of shells found on a coastline is usually limited. Here, however, there were many shells of different sizes, colors, and chronological positions in the long evolutionary history of these organisms. Why was this? I was told that Pleistocene marine deposits had been excavated in order to renew the sand on the beach. As a result, recent shells were mixed with older shells, which are usually darker in color. The top layer of sand is in an oxidizing environment, so it is light or reddish in color. The deeper layer is darker because it is in a reducing environment. The same thing happens with shells. As we continued our walk, we collected older shells. These were worn oyster shells, probably dating back to the Pliocene era, about five million years ago. Were they carried there by the Savio, Sillaro, or Po rivers? Time was no longer one-dimensional. Past and present coexisted. Our walk had turned into a shell collection, which had turned into a search for fossils, and all of this was perfectly normal.

It was normal because what is eroded in one place is always redeposited in another. It was extraordinary because I only realized it now. There were many recent and ancient fossils, but to my daughter, they were just shells with no distinguishing features. That's how they are for most people walking on the beach. It seemed impossible to me that no one else had noticed them, yet many of us were collecting those shells. Another thought crossed my mind: If all this was normal, then it had happened in the past and would happen again in the future. It's always difficult to assert something other than the evidence (or presumed evidence).



Rembrandt Harmenszoon van Rijn, *The shell (Conus marmoreus)*, 1650, etching, drypoint and burin, second state of three, 97 x 132 mm, Haarlem, Teylers Museum (www.wga.hu).

The vision

The challenge proposed here is a geometric interpretation of nature and, by extension, of the arts. Geometry is always present in natural forms, instilling certainty with its rhythms and patterns and becoming an integral part of the human cultural landscape. Since time immemorial, and increasingly as his productive capacities have been perfected, *homo faber* has surrounded himself with places and objects, both everyday and non-everyday (from fabrics to mosaics, from cabinetmaking to flooring, from tools to machines), populated with geometric figures, both instrumental and ornamental.

Every morning when we wake up, we cannot clearly distinguish our surroundings for a few moments. We perceive a dynamic mixture of light and dark without any meaningful context. The recognition of shapes begins two-dimensionally and only later extends to the third dimension, fully embracing shapes and objects. Gradually, figures emerge and become recognizable. In short, there is a brief but important period in which images are separated from their meaning. The brain must learn to correctly associate these images with

information from the sensory apparatus and pre-existing memories. This is how we transform perception into vision, defining and constructing the repertoire of shapes through which the world manifests itself.

In psychology, the ability to construct and modify perceived forms to adapt them to ideal forms stored in memory is known as *Gestalt*, a German term meaning "form," "structure," or "configuration." Our perception tends to interpret individual parts (such as points, lines, colors, and shading) as a complete image that does not necessarily correspond to a specific object but can be associated with other images based on empirically persuasive criteria. These processes are driven by a series of motivations that can be grouped under a few guiding headings, listed below. This list is designed to guide readers through the sequence of mathematical problems we will address in future lessons.

Proximity : we tend to see elements that are close together as parts of the same object, and those that are distant as parts of different objects;

Similarity : we interpret physically similar elements as parts of the same object, and physically different elements as parts of different objects.

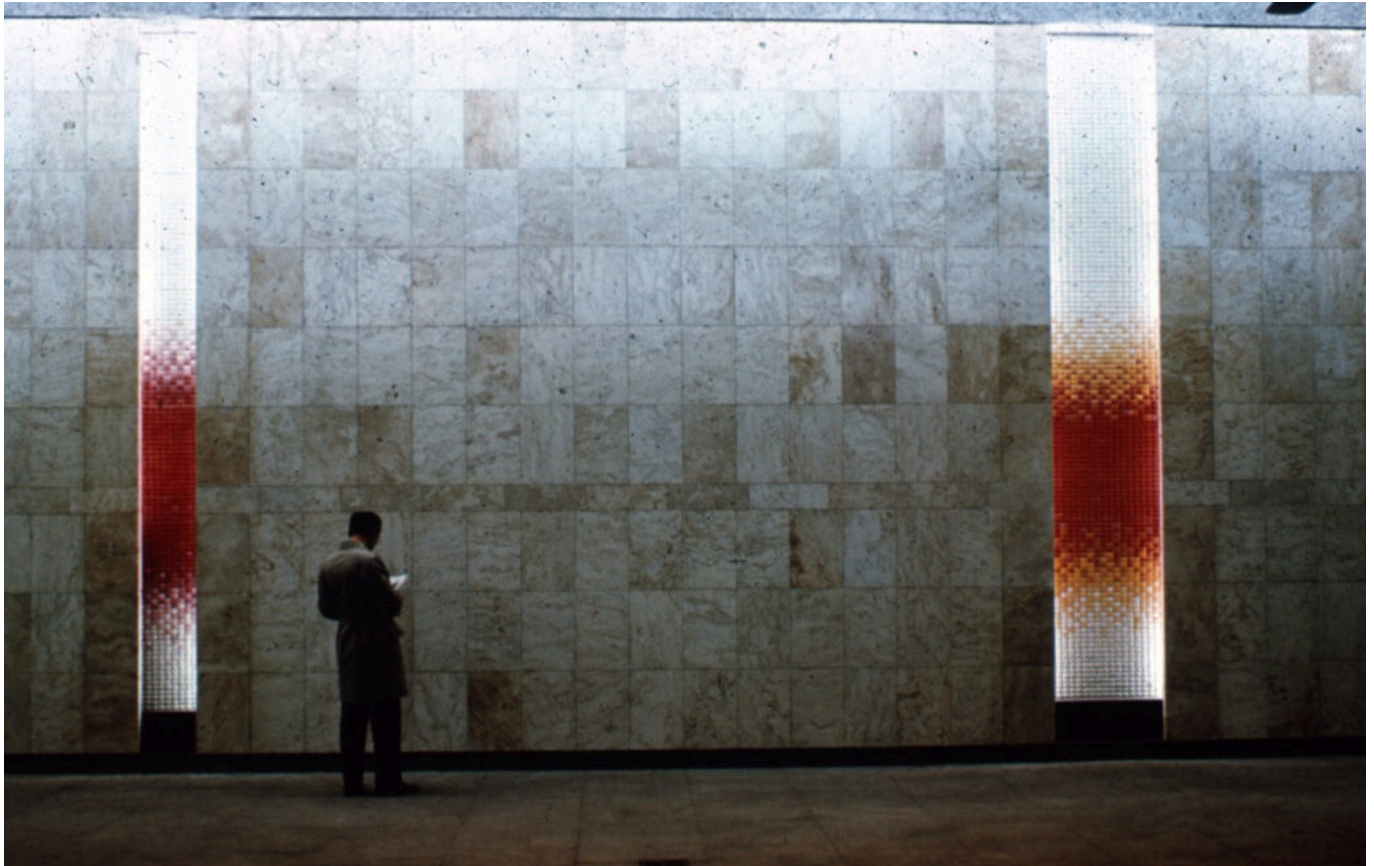
Closure : we imagine shapes as being bounded by a continuous edge and ignore any interruptions in that continuity, consequently perceiving shapes as complete even when they are partially hidden by other objects;

Continuity : when lines intersect, we tend to connect the different segments in such a way as to form lines that are as continuous as possible;

Common movement : when the elements that catch our attention move in the same direction and at the same speed, we imagine them as parts of the same object;

Formal harmony : our perception of things tends to be structured in the simplest, most orderly, symmetrical, and regular way possible.

All of these perceptual mechanisms have a tendency toward order, continuity, and symmetry rather than irregularity and disorder. These psychological processes also explain why we see illusory, pareidolic images when observing clouds in the sky or the alternation of full and empty spaces in a building. The search for and maintenance of meaningful forms that can generate meaning and guide behavior permeates every individual's life. External reality stimulates the production of images that affect perception of reality. Language is one way individuals make use of their perception of reality and compare it with that of others. Different human cultures are a distillation of this perception. In turn, science has a universal language in mathematical language, which knows no barriers, although it must be agreed that scientific views of the world are also interpretations of reality, and not reality itself.



Wojciech Fangor, *Mosaic inserts (detail)*, 1960-62, ceramic tiles, Warsaw, Śródmieście Railway Station (image from the magazine "Architektura," 3/1972, now Wikimedia).

Observation and experience

In the opening lines of the *Metaphysics*, Aristotle argues that wonder at the world around us is the driving force behind human inquiry. Human inquiry must constantly strive to eliminate wonder by developing theoretical knowledge that can explain the facts that gave rise to it. Once these facts are understood, we would be surprised if they turned out to be different from how they appear.

Humans interpret their surroundings through observation. Observation and its consolidation – experience – are the determining factors in making discoveries and inventions. Through observation, we can discern and compare seemingly incomparable events, objects, and living beings. Our ability to observe is influenced by various circumstances, such as similarity to known objects, proximity of similar elements, and identification of sequences. We tend to perceive coherent configurations of elements, isolating, grouping, and abstracting them. These processes play a decisive role in allowing us to modify the reality around us to our advantage. Many artifacts, from everyday objects to architectural elements and from medicines to dyes, are clearly replicas of what already exists in nature. This is one reason they are so effective and useful. Nature never relies on chance.

Throughout history, observation has been a starting point for moving beyond worldly appearances. Through the observer's lens, objects become mathematical entities. Mathematics is the complete expression of every methodical observation. It has always been the premier tool for describing our surroundings. This work is guided by the idea that form is a mathematically determinable essence. According to the approach presented here, mathematical-geometric representation becomes the representation of a construct that can be transcribed, disseminated, and replicated. Its most real and profound meaning lies here.

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Homepage: Philippe Wolfers, Civilization and Barbarism, 1897, silver, ivory and onyx, 46 x 67 x 27 cm, Bruxelles, Musées Royaux d'Art et d'Histoire (www.wga.hu).

Below: English manufacture, Marbled wallpaper, 1815–30, 71.5 x 57 cm, New York, Cooper Hewitt Smithsonian Design Museum.

